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FUZZY GOAL PROGRAMMING APPROACH FOR IDENTIFYING TARGET UNIT IN COMBINED-ORIENTED DEA MODELS: APPLICATION IN BANK INDUSTRY

Abstract: *In traditional data envelopment analysis (DEA) the projection of an inefficient unit onto efficient frontier is considered as an efficient target that can be used as a benchmark for inefficient units. The inefficient unit may to decrease large amounts of inputs and increase large amount of outputs in order to reach the target unit, which it may be impossible to do them all at the same time. On the other hand, the decision maker's preferences for finding the target unit are not taken into account in the conventional DEA models. In order to help an inefficient unit reach a target unit according to the decision maker's (DM's) preferences, an equivalent formulation between combined-oriented DEA model and multiple objective linear programming (MOLP) is established. A strategy based on goal programming is proposed to solve the resulting MOLP problem in order to consider the DM's preferences by considering the aspiration level for inputs and outputs. In contrast to existing approaches, the proposed strategy takes into account the preferences of DM without using any interactive MOLP method. The strategy presented is used in the evaluation of seven branches of British banks.*

Keywords: *Data envelopment analysis; Fuzzy goal programming; Multiple objective linear programming; Preferred target setting*

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric method based on linear programming. It measures the relative efficiency of a series of homogeneous decision-making units and improves their performances. This model is a useful tool for managing and making policy for decision-making units (DMUs). Different concepts of this method have been introduced during the past years and each of them is a useful method individually. In the original DEA models, the decision maker's (DM) view is ignored, the performance of each DMU is evaluated based on the observation and the DM's view has no role in the evaluations. Applying the DM's view, probably changes the improvement direction. Different methods have been suggested to incorporate the DM's view for the DMUs performance evaluation.

It is worthwhile to note that DEA target setting can be approached as a multi-objective linear programming (MOLP) problem. Joro et al. (1998) established that DEA and MOLP are structurally identical. Lins et al. (2004) proposed the MOLP model for target optimisation which directly optimises the target inputs and outputs instead of their corresponding multiplicative ratios. Lozano and Villa (2010) proposed a strategy of gradual improvements with successive intermediate targets in order to help an inefficient unit reach a distant target. Wong et al. (2009) established an equivalence model between DEA and MOLP and explored how a DEA model can be solved by various interactive multi-objective models. Yang et al. (2009) investigated three equivalence models between the output-oriented dual DEA model and the minimax reference point formulations to take into account the decision makers' preferences in an interactive manner. Malekmohammadi et al. (2010) improved these formulations to obtain models that address both inputs and outputs in order to decrease total input consumption and increase total output production. Lotfi et al. (2010) have used Zionts–Wallenius method to reflect the DM's preferences in the process of assessing efficiency in the output-oriented and the general combined-oriented DEA models. Yang et al. (2010) developed a hybrid minimax reference point-DEA approach to incorporate the value to search for the most preferred solution along the efficient frontier for each DUM. Yang et al. (2012) explored graphical and analytical procedures for generating efficient frontiers for multiple DEA models. Razipour-GhalehJough et al. (2020) proposed a new approach for target setting in the presence of weight restrictions. Ghazi et al. (2020) suggested a pure mathematical procedure for target setting in the presence of negative data based on MOLP methodology. Gutiérrez and Lozano (2016) proposed a multi-objective DEA model to explore the possible trade-offs in the output Pareto efficient frontier of a given airport in efficiency assessment of European small and medium sized airports. Lozano and Soltani (2018) proposed the lexicographic directional distance function approach for DEA target setting. Khalafi (2021) proposed a non-radial inefficiency in terms of Russell model based on an interactive approach to gain the appropriate operational benchmark.

This paper proves the equivalency between the combined-oriented DEA model and MOLP and then proposes a method based on fuzzy goal programming to involve the DM's preferences in the evaluation of DMUs. Finally a new efficiency is proposed and its relationship with the efficiency of classic DEA model is explained. In contrast the above-mentioned existing method, the proposed approach finds the target unit for each inefficient unit without using any interactive MOLP method.

The reminder of this paper is organized as follows. In Section 2, Mohamed's approach is first extended to consider several objective functions and then a fuzzy goal programming is suggested for solving MOLP. The equivalence of the combined-oriented DEA model and MOLP is established in Section 3. DM's views, accordingly, can be applied on the inputs and outputs simultaneously. In Section 4, a method based on fuzzy concept for evaluation and target setting will be offered in which the DM's views can be applied on the performance evaluation of DMUs. The proposed approach is used to evaluate the performance of seven branches of UK bank in Section 5. Finally, Section 6 gives concluding remarks.

2. Fuzzy multiple objective linear programming

In this section, the goal programming (GP) approach to fuzzy programming problems introduced by Mohamed (1997) is extended to solve MOLP problems. In the GP model formulation, first the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then the achievement of the highest membership value to the extent possible of each of the fuzzy goals is considered.

Let there be a MOLP with p objective function in the following form:

$$\begin{aligned} & Max\left[z_1(x), z_2(x), ..., z_p(x)\right] \\ & s.t \\ & Ax \precsim b \\ & x \geq 0 \end{aligned} \qquad (1)$$

where A is a $(m \times n)$ matrix and $\precsim$ is the fuzzy form of $\leq$ that to be understood essentially less than.

Let $g = (g_{1,}g_{2,...,}g_p)$ be an aspiration level for objective function. In this case, we seek a solution that: 1) the objective function will be more than its aspiration level in fuzzy environment and 2) the constraint are considered in the best fuzzy condition. Considering Mohamed's approach the following model is resulted for solving MOLP model (1):

$$
\begin{aligned}
& Min \sum_{k=1}^{p} d_{1k}^{-} + \sum_{i=1}^{m} d_{2i}^{-} \\
& s.t \\
& \frac{z_k(x) - z_k^{-}}{g_k - z_k^{-}} + d_{1k}^{-} - d_{1k}^{+} = 1 \qquad k = 1,2,\ldots,p \\
& \frac{b_i^{+} - a_i x}{b_i^{+} - b_i} + d_{2i}^{-} - d_{2i}^{+} = 1 \qquad i = 1,2,\ldots,m \\
& d_{1k}^{-}.d_{1k}^{+} = 0 \\
& d_{2i}^{-}.d_{2i}^{+} = 0 \\
& d_{1k}^{-}, d_{1k}^{+} \geq 0 \\
& d_{2i}^{-}, d_{2i}^{+} \geq 0 \\
& x \geq 0 \\
& k = 1,2,\ldots,p \ , \ \ i = 1,2,\ldots,m
\end{aligned} \tag{2}
$$

3. Equivalences between MOLP and combined-oriented DEA model

DEA and MOLP are two useful tools in managing affairs that the decision maker can use them in control and programming.

3.1. DEA models

DEA is a method for assessing the productivity efficiency of DMUs such as the branch of bank, hospitals, schools and etc. that they consume some of resources (input) to product the same kind of services (output). DEA after the examination of all DMUs and specifying the extent of their efficiency, determines a virtual unit for inefficient units to improve their performance (Charnes et al. 1990; Banker et al. 1984).

To find a unit with less input to produce output equal of DMU_{j_0} one can to find a DMU with greater output and the input equal of DMU_{j_0} one. Consider the following DEA models:

$$
\begin{aligned}
& Min\,\theta_{j_0} \\
& s.t \\
& X\lambda - \theta_{j_0} x_{j_0} \leq 0 \\
& y_{j_0} - Y\lambda \leq 0 \\
& \alpha e^t \lambda = \alpha \\
& \lambda \geq 0
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& Max\,\phi_{j_0} \\
& s.t \\
& X\lambda - x_{j_0} \le 0 \\
& \phi_{j_0} y_{j_0} - Y\lambda \le 0 \\
& \alpha\, e^t \lambda = \alpha \\
& \lambda \ge 0
\end{aligned}
\tag{4}
$$

where X is an $m \times n$ input matrix and Y is an $s \times n$ output. If α=0 then models (3) and (4) are respectively CCR input and output oriented models and if α=1 then models (3), (4) are respectively BCC input and output oriented models.

Model (3) wants to decrease inputs and model (4) wants to increase outputs. The aim of the following combined-oriented DEA model is to search for a DMU with more output and less input on efficiency frontier.

$$
\begin{aligned}
& Max\;\theta_{j_0} \\
& s.t \\
& X\lambda \le (1-\theta_{j_0}) x_{j_0} \\
& Y\lambda \ge (1+\theta_{j_0}) y_{j_0} \\
& \alpha\, e^t \lambda = \alpha \\
& \lambda \ge 0
\end{aligned}
\tag{5}
$$

Definition 1: DMU_{j_0} is said to be strongly efficient in combined-oriented DEA model (5) if only if $\theta_{j_0}^{\,*} = 1$ and all of slack and surplus variables be zero (in optimal solution).

3.2. Combined DEA model as a MOLP

In original models of DEA, the examination of efficiency of each unit is performed in the input orientated and in the output orientated. Considering the two views, it is possible to consider DEA as a MCDM. Wong et al. (2009) showed the equivalence of the CCR model in output orientated and MOLP. But considering the nature of their model, the views are only applied in outputs. In this section, the equivalence of MOLP and the combined model is proved and in the next section, a method based on the FGP will be presented to apply the DM's view on both inputs and outputs.

Consider the following MOLP in general way:

$$\begin{aligned} & Max[\, f_1(\lambda),......., f_k(\lambda)] \\ & s.t \\ & \lambda \in S \end{aligned} \tag{6}$$

Definition 2: Suppose λ^* is a solution of model (6) so it is strongly efficient if there is no $\lambda \in S$ that $(f_1(\lambda),......., f_k(\lambda)) \geq (f_1(\lambda^*),......., f_k(\lambda^*))$ and $(f_1(\lambda),......., f_k(\lambda)) \neq (\, f_1(\lambda^*),......., f_k(\lambda^*))$. If a solution is not efficient then we say it is inefficient.

Considering an aspiration level for t^{th} objective (f_t^*) and weighting index w for t^{th} objective function the, following model minimizes the maximum weighted derivation of each objective function (Yang et al. 200):

$$\begin{aligned} & \underset{\lambda}{Min} \quad \underset{1 \leq t \leq k}{Max}\{w_t(\, f_t^* \; - f_t(\lambda))\} \\ & s.t \\ & \lambda \in S \end{aligned} \tag{7}$$

By use of an auxiliary variable the above model can be rewritten as follow (Lightner and Director 1981, Yang and Li 2002)

$$\begin{aligned} & Min \quad \theta \\ & s.t \\ & w_t(\, f_t^* - f_t(\lambda)) \leq \theta \quad t = 1,2,...,k \\ & \lambda \in S \end{aligned} \tag{8}$$

Consider we have m+s objective function (k=m+s) and $S = \left\{ \lambda \middle| \alpha e^t \lambda = \alpha, \lambda_j \geq 0, j = 1,2,...,n \right\}$ in model (6) now we define:

$$f^I{}_i(\lambda) = \sum_{j=1}^{n} \lambda'_j \, x_{ij} \quad i = 1, 2, \ldots, n \tag{9}$$

$$f^O{}_r(\lambda) = \sum_{j=1}^{n} \lambda''_j \, y_{rj} \quad i = 1, 2, \ldots, n \tag{10}$$

where:

$$\lambda'_j = \begin{cases} -\lambda_j & j \neq j_0 \\ 1 - \lambda_j & otherwise \end{cases} \qquad \lambda''_j = \begin{cases} \lambda_j & j \neq j_0 \\ 1 + \lambda_j & otherwise \end{cases} \tag{11}$$

Now fix:

$$\bar{f}^{O}_{rj_0} = f_r(\lambda''^{\,r}) \quad r = 1,2,\dots,s \tag{12}$$

where $\lambda'^{\,i}$ and $\lambda''^{\,r}$ are respectively optimum solutions of models (13) and (14) as follows:

$$\begin{aligned} &Min\ f^{I}_{i}(\lambda) = \sum_{j=1}^{n} \lambda_j x_{ij} \\ &s.t \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rj_0} \\ &\lambda_j \ge 0 \end{aligned} \tag{13}$$

$$\begin{aligned} &Max\ f^{O}_{r}(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} \\ &s.t \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ij_0} \\ &\lambda_j \ge 0 \end{aligned} \tag{14}$$

Now consider the following relations:

$$w^{o}{}_{r} = \frac{1}{y_{rj_0}} \qquad r = 1,2,\dots,s \tag{15}$$

$$w^{I}{}_{i} = \frac{1}{x_{ij_0}} \qquad i = 1,2,\dots,m \tag{16}$$

$$F^{\min} = \min_{1\le i\le m}\{w_i^{I}\, \bar{f}^{I}_{ij_0}\} = \min_{1\le i\le m}\{\frac{\bar{f}^{I}_{ij_0}}{x_{ij_0}}\} \tag{17}$$

$$F^{\max} = \max_{1\le r\le s}\{w_r^{o}\, \bar{f}^{O}_{rj_0}\} = \max_{1\le r\le s}\{\frac{\bar{f}^{O}_{rj_0}}{y_{rj_0}} \tag{18}$$

$$f_i^{*I} = \frac{F^{\min}}{w_i^{I}} = x_{ij_0} F^{\min} \tag{19}$$

$$f_r^{*O} = \frac{F^{\max}}{w_r^{o}} = y_{rj_0} F^{\max} \tag{20}$$

$$F = \max\{F^{\max}, F^{\min}\} \tag{21}$$

$$\theta = F - \theta_{j_0} \tag{22}$$

Theorem 1: Let $k=m+s$ and $S=\left\{\lambda \middle| \alpha e^t\lambda=\alpha, \lambda_j \geq 0, j=1,2,...,n\right\}$ in model (8) with substituting (9)-(12) and (15)-(22), combined-oriented DEA model (5) and MOLP model (8) are equivalent.

Proof: Consider the constraint of model (5)

$$\sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{ij_0}(1-\theta_{j_0}) \Leftrightarrow \sum_{j=1}^{n}\lambda_j x_{ij} - x_{ij_0} + \theta_{j_0} x_{ij_0} \leq 0$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{rj_0}(1+\theta_{j_0}) \Leftrightarrow -\sum_{j=1}^{n}\lambda_j y_{rj} + y_{rj_0} + y_{rj_0}\theta_{j_0} \leq 0$$

$$\theta_{j_0} x_{ij_0} - \sum_{j=1}^{n}\lambda'_j x_{ij} \leq 0 \Leftrightarrow \frac{1}{w^I{}_i}\theta_{j_0} - f_i(\lambda') \leq 0,\ \theta_{j_0} y_{rj_0} - \sum_{j=1}^{n}\lambda''_j x_{ij} \leq 0 \Leftrightarrow \frac{1}{w^O{}_r}\theta_{j_0} - f_r(\lambda'') \leq 0$$

$$-w^I{}_i f_i(\lambda') \leq -\theta_{j_0} \Leftrightarrow F^{\min} - w^I{}_i f^I{}_i(\lambda) \leq F^{\min} - \theta_{j_0} \leq F - \theta_{j_0}$$

$$-w^O{}_r f_r(\lambda'') \leq -\theta_{j_0} \Leftrightarrow F^{\max} - w^O{}_r f^O{}_r(\lambda) \leq F^{\max} - \theta_{j_0} \leq F - \theta_{j_0}$$

$$w_i(\frac{F^{\min}}{w_i} - f^I{}_i(\lambda)) \leq \theta \Leftrightarrow w_i(f^*{}_i - f^I{}_i(\lambda)) \leq \theta, w_t(\frac{F^{\max}}{w_r} - f^O{}_r(\lambda)) \leq \theta \Leftrightarrow w_r(f^*{}_r - f^I{}_r(\lambda)) \leq \theta$$

So the constraints of two models are equal.

In the other hand for objective functions:

$$Min\,\theta \Leftrightarrow Min(F-\theta_{j_0}) \Leftrightarrow Min-\theta_{j_0} \Leftrightarrow Max\,\theta_{j_0}.$$

Therefore according to equations (9)-(12) and (15)-(22), the combined-oriented DEA model (5) is equivalent with model (8). ■

Thus combined-oriented DEA model (5) can be rewritten as the following model:

$$\begin{aligned}&Max[f^O{}_1(\lambda), f^O{}_2(\lambda),..., f^O{}_s(\lambda), f^I{}_1(\lambda), f^I{}_2(\lambda),..., f^I{}_m(\lambda)]\\&s.t\\&\lambda \in S\end{aligned} \qquad (23)$$

The above analysis show that combined DEA model is actually constructed to locate a specific efficient solution, termed as DEA efficient solution on the efficient frontier of the two following generic MOLP formulation for observed DMU_{j_0}:

$$Max[\sum_{j=1}^{n}\lambda_j'' y_{1j}, \sum_{j=1}^{n}\lambda_j'' y_{2j} ..., \sum_{j=1}^{n}\lambda_j'' y_{rj}, \sum_{j=1}^{n}\lambda_j' x_{1j}, \sum_{j=1}^{n}\lambda_j' x_{2j}, ..., \sum_{j=1}^{n}\lambda_j' x_{mj}]$$
$$s.t \qquad (24)$$
$$\lambda \in S$$

$$Max[\sum_{j=1}^{n}\lambda_j y_{1j} + y_{1j_0}, ..., \sum_{j=1}^{n}\lambda_j y_{sj} + y_{sj_0}, -\sum_{j=1}^{n}\lambda_j x_{1j} - x_{1j_0}, ..., -\sum_{j=1}^{n}\lambda_j x_{mj} - x_{mj_0}]$$
$$s.t \qquad (25)$$
$$\lambda \in S$$

Notice that in order to interpret this model; DEA seeks a virtual DMU which produces more output or equal to DMU_{j_0} for evaluation of a DMU. This model seeks a DMU which produces output more than or equal to DMU_{j_0} with the minimum input.

4. A FGP procedure for preferred target setting

In Section 3, the equivalence of combined-oriented DEA model and MOLP has been established. With a procedure based on discussions of Sections 2 and 3 for finding optimal solution model in constant return to scale (CRS) model (22), it is sufficient to solve the following model:

$$Min \sum_{i=1}^{m} d_{1i}^{-} + \sum_{r=1}^{s} d_{2r}^{-}$$
$$s.t \qquad (26)$$
$$\frac{z_i^{+} - \sum_{j=1}^{n}\lambda_j x_{ij}}{z_i^{+} - \bar{x}_i} + d_{1i}^{-} - d_{1i}^{+} = 1 \qquad i = 1,2,...,m$$
$$\frac{\sum_{j=1}^{n}\lambda_j y_{rj} - z_r^{-}}{\bar{y}_r - z_r^{-}} + d_{2r}^{-} - d_{2r}^{+} = 1 \quad r = 1,2,...,s$$

$$\lambda \ge 0 \tag{26}$$
$$d_{1i}^{-}.d_{1i}^{+} = 0$$
$$d_{2r}^{-}.d_{2r}^{+} = 0$$
$$d_{1i}^{-}, d_{1i}^{+} \ge 0$$
$$d_{2r}^{-}, d_{2r}^{+} \ge 0$$
$$r = 1,2,...,s, \;\; i = 1,2,...,m$$

In which z_r^{-} is the minimum acceptable value for the r-th output and z_i^{+} is the maximum acceptable value for i-th input and $\overline{x_i}$, $\overline{y_r}$ are aspiration levels of i-th input and r-th output of DMU0 respectively.

Definition 2: DMU_{j_0} is strongly efficient in model (26) if and if for any optimal solution $(\lambda^*, d_{1i}^{-*}, d_{1i}^{+*}, d_{2r}^{-*}, d_{2r}^{+*})$ we have:

$$d_{1i}^{-*} = d_{1i}^{+*} = 0 \quad \forall i = 1, 2, ..., m$$
$$d_{2r}^{-*} = d_{2r}^{+*} = 0 \quad \forall r = 1, 2, ..., s$$

Theorem 2: Let (λ^*, θ^*) be the optimal solution of combined-oriented DEA model. Considering aspiration levels in model (26) from optimal solution model (5), $(\lambda^*, 0, d_{1i}^{+*}, 0, d_{2r}^{+*})$ is the optimal solution of model (26) where:

$$d_{1i}^{+*} = \frac{z_i^{+} - \sum_{j=1}^{n} \lambda_j^{*} x_{ij}}{z_i^{+} - \overline{x}_i} - 1 \qquad d_{2r}^{+*} = \frac{\sum_{j=1}^{n} \lambda_j^{*} y_{rj} - z_r^{-}}{\overline{y}_r - z_r^{-}} - 1$$

Proof: Let (λ^*, θ^*) be optimal solution of combined-oriented DEA model.

Thus

$$\sum_{j=1}^{n} \lambda_j^{*} x_{ij} \le (1-\theta^*) x_{i0} = \overline{x}_i \Rightarrow -\sum_{j=1}^{n} \lambda_j^{*} x_{ij} \ge -\overline{x}_i \Rightarrow z_i^{+} - \sum_{j=1}^{n} \lambda_j^{*} x_{ij} \ge z_i^{+} - \overline{x}_i$$

$$\frac{z_i^{+} - \sum_{j=1}^{n} \lambda_j^{*} x_{ij}}{z_i^{+} - \overline{x}_i} \ge 1, \quad (z_i^{+} > \overline{x}_i)$$

In the other hand $\frac{z_i^+ - \sum_{j=1}^{n} \lambda_j^* x_{ij}}{z_i^+ - \bar{x}_i} + d_{1i}^- - d_{1i}^+ = 1$. If there exists a $1 \le t \le n$ in which $d_{1t}^- > 0$ then $\frac{z_i^+ - \sum_{j=1}^{n} \lambda_j^* x_{ij}}{z_i^+ - \bar{x}_i} = 1 - d_{1i}^- < 1$. And it is in contradiction with $\frac{z_i^+ - \sum_{j=1}^{n} \lambda_j^* x_{ij}}{z_i^+ - \bar{x}_i} \ge 1$. Thus $d_{1t}^- = 0$. With a same procedure, it can be shown that $d_{2r}^- = 0$. ■

Theorem 3: Considering optimal solution model (5) as aspiration level, DMU_{j_0} is strongly efficient in combined-oriented DEA model if and only if DMU0 is strongly efficient in model (26).

Proof: (only if part) let DMU_{j_0} is strongly efficient in (5). Negate that is inefficient in (26) let $(\lambda^*, d_{1i}^{-*}, d_{1i}^{+*}, d_{2r}^{-*}, d_{2r}^{+*})$ is an optimal solution of (26). As Theorem 2 it is clear that $d_{1i}^{-*} = 0$, $d_{2r}^{-*} = 0$ so without distortion of the integrity of problem suppose there exit a $1 \le t \le m$ that

$$d_{1t}^{-*} > 0 \Rightarrow \frac{z_t^+ - \sum_{j=1}^{n} \lambda_j^* x_{tj}}{z_t^+ - \bar{x}_t} - 1 > 0 \Rightarrow \frac{z_t^+ - \sum_{j=1}^{n} \lambda_j^* x_{tj}}{z_t^+ - \bar{x}_t} > 1 \Rightarrow z_t^+ - \sum_{j=1}^{n} \lambda_j^* x_{tj} > z_t^+ - \bar{x}_t$$

$$\Rightarrow -\sum_{j=1}^{n} \lambda_j^* x_{tj} > -\bar{x}_t \Rightarrow \sum_{j=1}^{n} \lambda_j^* x_{tj} < \bar{x}_t = (1-\theta^*) x_{tj_0} < x_{tj_0}$$

Thus $(\lambda^*, \theta^*, s^{-*}, s^{+*})$ is an optimal solution of combined-oriented DEA model (5) where $\theta^* = 0$, $s_t^{-*} > 0$ and it is contradiction with efficiency DMU_{j_0} for model (5).

(if part) Let DMU_{j_0} be strongly efficient in (26) and $(\lambda^*, \theta^*, s^{-*}, s^{+*})$ be an optimal solution of model (5). Suppose DMU_{j_0} is inefficient in (5). Without loss of generality, let there is $1 \le t \le m$ that $s_t^- > 0$ thus:

$$\sum_{j=1}^{n} \lambda_j^* x_{tj} < (1-\theta^*) x_{tj_0} = \bar{x}_t \Rightarrow -\sum_{j=1}^{n} \lambda_j^* x_{tj} > -\bar{x}_t \Rightarrow z_t^+ - \sum_{j=1}^{n} \lambda_j^* x_{tj} > z_t^+ - \bar{x}_t \Rightarrow$$

$$z_t^{+}-\sum_{j=1}^{n}\lambda_j^{*}x_{tj} > z_t^{+}-\bar{x}_t \Rightarrow \frac{z_t^{+}-\sum_{j=1}^{n}\lambda_j^{*}x_{tj}}{z_t^{+}-\bar{x}_t} > 1 \Rightarrow (z_t^{+}-\bar{x}_t > 0) \Rightarrow d_{1t}^{-^{*}} = 0\ ,\ d_{1t}^{+^{*}} > 0\ .$$

So $(\lambda^{*}, d_{1i}^{-^{*}}, d_{1i}^{+^{*}}, d_{2r}^{-^{*}}, d_{2r}^{+^{*}})$ is an optimal solution that $d_{1t}^{+^{*}} > 0\ \ (1 \le t \le m)$. This is in contradiction with strongly efficiency in (26).□

Theorem 4: Considering optimal solution model (5) as an aspiration level, DMU_{j_0} is strongly efficient in combined-oriented DEA model if and only if DMU_{j_0} is strongly efficient in (6).

Proof: (Only if part) let DMU_{j_0} be strongly efficient in (5). Thus, if $(\lambda^{*}, \theta^{*}, s^{-^{*}}, s^{+^{*}})$ is optimal solution of (6) then $\theta^{*} = 0,\ s^{-^{*}} = 0,\ s^{+^{*}} = 0$.

By contradiction assume that DMU_{j_0} is inefficient in (6). Thus there is $(\bar{x}, \bar{y})$ that $(-x^{*}, y^{*}) \le (-\bar{x}, \bar{y})$, $(-x^{*}, y^{*}) \ne (-\bar{x}, \bar{y})$ where:

$$x_i^{*} = \sum_{j=1}^{n}\lambda_j^{*}x_{ij} + x_{ij_0}\ ,\quad \bar{x}_i = \sum_{j=1}^{n}\bar{\lambda}_j x_{ij} + x_{ij_0} \qquad 1 \le i \le m$$

$$y_r^{*} = \sum_{j=1}^{n}\lambda_j^{*}y_{rj} + y_{rj_0}\ ,\quad \bar{y}_r = \sum_{j=1}^{n}\bar{\lambda}_j y_{rj} + y_{rj_0}\ , \qquad 1 \le r \le s$$

Without loss of generality, assume

$$\sum_{j=1}^{n}\bar{\lambda}_j x_{tj} + x_{tj_0} < \sum_{j=1}^{n}\lambda_j^{*}x_{tj} + x_{tj_0} \Rightarrow \sum_{j=1}^{n}\bar{\lambda}_j x_{tj} < \sum_{j=1}^{n}\lambda_j^{*}x_{tj} = x_{tj_0}(1-\theta^{*}) = x_{tj_0}\ (\theta^{*}=0)$$

$$\Rightarrow \sum_{j=1}^{n}\bar{\lambda}_j x_{tj} < \sum_{j=1}^{n}\lambda_j^{*}x_{tj} = x_{tj_0}(1-\theta^{*}) = x_{tj_0}\ \ (\theta^{*}=0)$$

$$\Rightarrow \sum_{j=1}^{n}\bar{\lambda}_j x_{tj} < x_{tj_0} \Rightarrow \exists\alpha > 0\ \ \sum_{j=1}^{n}\bar{\lambda}_j x_{tj} + \alpha x_{tj_0} = x_{tj_0} \Rightarrow \sum_{j=1}^{n}\bar{\lambda}_j x_{tj} = x_{tj_0}(1-\alpha)$$

Thus $\left(\bar{\lambda}, \bar{\theta}, \overline{s^{-}}, \overline{s^{+}}\right)$ is a feasible solution of (5) where:

$$\overline{s_i^{-}} = x_{ij_0}(1-\alpha) - \sum_{j=1}^{n}\bar{\lambda}_j x_{ij}, 1 \le i \le m\ ,\ i \ne t,\ \ \overline{s_t^{-}} = 0\ \ ,\ \ \overline{s_r^{+}} = \sum_{j=1}^{n}\bar{\lambda}_j y_{rj} - y_{rj_0}(1+\alpha), 1 \le r \le s$$

And $\bar{\theta} = \alpha > 0 = \theta^{*}$ that it contradicts with optimality of $\left(\lambda^{*}, \theta^{*}, s^{-^{*}}, s^{+^{*}}\right)$.

(If part) conversely suppose DMU_{j_0} is efficient in (6) and it is inefficient in combined-oriented DEA model. Suppose $\left(\lambda^*, \theta^*, s^{-^*}, s^{+^*}\right)$ is an optimal solution of the combined-oriented DEA model. There exit $1 \le t \le m$ that $s_t^- > 0$.

Thus, $\sum_{j=1}^{n} \lambda_j^* x_{tj} < (1-\theta^*) x_{tj_0} < x_{tj_0}$.

Now suppose $x_i^* = \sum_{j=1}^{n} \lambda_j^* x_{ij} \quad 1 \le i \le m$ and $y_r^* = \sum_{j=1}^{n} \lambda_j^* y_{rj} \quad 1 \le r \le s$.

Thus $(-x^*, y^*) \le (-x_{j_0}, y_{j_0})$ and $(-x^*, y^*) \ne (-x_{j_0}, y_{j_0})$.

This is contradiction with efficiency of DMU_{j_0} in (6). Hence DMU_{j_0} is strongly efficient in combined-oriented DEA model.

5. An application

Our method is illustrated here via an application with real-world data. Yang .et al.'s (2012) data is used to evaluate the performance of seven branches of UK bank which includes Abbey National, Barclays, Halifax, HSBC, Lloyds TSB, NatWest and RBS. In this evaluation, three inputs (Number of branches, Number of ATM, Number of staff) and three outputs (Total revenue, Corporate image, Customer satisfaction) were considered for each branch (Table 1).

Table 1: Data set of the UK retail banks

DMU	Bank	Inputs			Outputs		
		No. of branches	No. of ATMs	No. of staff	Total revenue	Corporate image*	Customer satisfaction*
				('000)	('000)	('0,000)	(£m)
1	Abbey Nat.	2.00	2.18	2.35	10.57	3.40	6.79
2	Barclays	1.95	3.19	8.43	13.35	6.66	2.55
3	Halifax	0.80	2.10	3.21	8.14	1.92	9.17
4	HSBC	1.75	4.00	13.30	23.67	8.47	5.82
5	Lloyds TSB	2.50	4.30	9.27	14.01	3.44	6.57
6	NatWest	1.73	3.30	7.70	12.04	2.53	4.86
7	RBS	0.65	1.53	2.67	7.36	1.26	7.28

*Corporate image and customer satisfaction values are converted scores based on the average expected utility of survey respondents.

The results obtained from the evaluation of branches by the combined model of DEA are shown in Table 2. It reveals that branches of Lioyds and Natwest are inefficient.

Table 2: Combined efficiency results

Observed DMU's composite unit									
DMU	Bank	Efficiency	1	2	3	4	5	6	7
1	Abbey Nat.	1.00	1.00						
2	Barclays	1.00		1.00					
3	Halifax	1.00			1.00				
4	HSBC	1.00				1.00			
5	Lloyds TS	0.23	0.29			0.28			1.02
6	NatWest	0.19	0.23			0.30			0.62
7	RBS	1.00							1.00

The performance of different parts and the direct of improvement for NatWest are shown in Table 3.

Table 3: Combined efficiency results of NatWest

Performance	Inputs			Outputs		
	No. of branches	No. of ATMs	No. of staff	Total revenue	Corporate image	Customer satisfaction
	('000)	('000)	('0000)	(£m)		
Evaluated unit	1.73	3.30	7.70	12.04	2.53	4.86
Composite unit	1.4	2.68	6.26	14.28	4.16	7.92
Improvement	−0.33	−0.62	−1.44	2.24	1.63	3.06

It is transparent that, for example, the first input is reduced from 1.73 to 1.4. The second input is reduced from 3.3to2.68 and third input is decreased from 7.7 to 6.26. So, the amounts of increasing of outputs are respectively: 2.24, 1.63 and 3.06. In examination of these branches by proposed model, the maximum acceptable value for the input z_i^+ is considered equal of DMU0 input ($z_i^+ = x_{ij_0}$). The minimum acceptable value for the output z_r^- is considered equal to DMU0 output ($z_r^- = y_{rj_0}$). In the first stage, aspiration level of proposed model is selected out of the optimal solution of combined model.

The following results are obtained and can be used to compare this model and the combined model. Examining by the combined model, it was shown that the branch number 1, 2, 3, 4, 7 were efficient. Moreover these branches are efficient based on proposed model. But the branches number 5 (Lloyds TSB) and 6 (NatWest) are inefficient in this model as they weren't in combined model too. The direct of improvement of the performance of the Natwest by the proposed model is presented in Table 4.

Table 4: Preferred target setting results of NatWest with optimal solution combined model as an aspiration levels

Performance	Inputs			Outputs		
	No. of branches	No. of ATMs	No. of staff	Total revenue	Corporate image	Customer satisfaction
		('000)	('000)	('0000)	(£m)	
Evaluated unit	1.73	3.30	7.70	2.04	2.53	4.86
Composite unit	1.4	2.68	5.70	13.09	4.08	7.91
Improvement	−0.33	−0.62	−2.00	1.05	1.55	3.05

In the next stage, the outputs which were confirmed by the manager in the evaluation of this branch in Wong .et al.'s research have been used as the aspiration level for each of outputs. The results of evaluation are shown in Table 5. By this evaluation, it is transparent that outputs 1, 2, 3 increase 11.5, 7.1, 7.07. Aspiration level of the previous chapter is used in this evaluation.

Table 5: Preferred target setting results of NatWest with confirmed output of Wong's research as an aspiration levels

Performance	Inputs			Outputs		
	No. of branches	No. of ATMs	No. of staff	Total revenue	Corporate image	Customer satisfaction
	('000)	('000)	('0000)		(£m)	
Evaluated unit	1.73	3.30	7.70	12.04	2.53	4.86
Composite unit	1.4	3.26	6.26	11.40	7.09	7.07
Improvement	−0.33	−0.04	−1.44	−0.64	4.56	2.21

As stated, in this evaluation the application of the DM's views and fulfilling his/her wishes is performed on outputs; but in a decision making process, the manager is compelled to apply some views on both the inputs and outputs to achieve the objects. It was performed by proposed model. The manager's wishes about the outputs were chosen like the previous stage evaluation from the manger's approved outputs; and the extent of DM's favored inputs of is 1.4, 2.5, and 6.2 for 1-th input 2-th input 3-th input respectively. Result of this preferred target setting is shown in Table 6.

Table 6: Preferred target setting results of NatWest with DM's wishes as an aspiration levels

Performance	Inputs			Outputs		
	No. of branches	No. of ATMs	No. of staff	Total revenue	Corporate image	Customer satisfaction
	('000)	('000)	('0000)	(£m)		
Evaluated unit	1.73	3.30	7.70	12.04	2.53	4.86
Composite unit	1.4	2.50	6.20	13.40	4.52	5.80
Improvement	−0.33	−0.80	−1.50	−1.36	1.99	0.94

6. Conclusions

In this paper, the equivalence of the combined model of DEA and MOLP has been explained. The method presented in this paper described the impacts of environmental factors which are actually uncertain parameters on the evaluation of the performance of decision making units with the help of FGP to evaluate the performance of the decision making units. Considering the aspiration level for each of them, so, the DM's ideals have applied on inputs and outputs in a same time. These factors are not applied on classic DEA. But they are important in efficiency of DMUs. The presented concepts are used in the evaluation of seven branches of British banks. In the first stage, considering the aspiration level for inputs and outputs from the combined model, it was shown that branch numbers 5 and 6 which were inefficient in combined model, are inefficient in this model too. In the second stage, the DM's view was applied just on outputs. On the third stage, considering the DM's preferences on input and output data, the evaluation and target setting has been performed. Generalization of the proposed approach for solving DEA models in the presence of undesirable factors (Nemati and Kazemi Matin 2019) and for finding target unit in network DEA (Yadollahi and Kazemi Matin 2022) can be interesting research works for the next studies.

Data Availability: No data was used to support this study.
Compliance with ethical standards
Conflict of interest: All the authors declare that they have no conflict of interest.
Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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